## Lesson 8: Similarity

## Classwork

## Example 1

In the picture below, we have a triangle $A B C$ that has been dilated from center $O$ by a scale factor of $r=\frac{1}{2}$. It is noted by $A^{\prime} B^{\prime} C^{\prime}$. We also have triangle $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$, which is congruent to triangle $A^{\prime} B^{\prime} C^{\prime}$ (i.e., $\Delta A^{\prime} B^{\prime} C^{\prime} \cong \triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ ).


Describe the sequence that would map triangle $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ onto triangle $A B C$.

## Exercises

1. Triangle $A B C$ was dilated from center $O$ by scale factor $r=\frac{1}{2}$. The dilated triangle is noted by $A^{\prime} B^{\prime} C^{\prime}$. Another triangle $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ is congruent to triangle $A^{\prime} B^{\prime} C^{\prime}$ (i.e., $\Delta A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} \cong \triangle A^{\prime} B^{\prime} C^{\prime}$ ). Describe a dilation followed by the basic rigid motion that would map triangle $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ onto triangle $A B C$.

2. Describe a sequence that would show $\triangle A B C \sim \triangle A^{\prime} B^{\prime} C^{\prime}$.

3. Are the two triangles shown below similar? If so, describe a sequence that would prove $\triangle A B C \sim \triangle A^{\prime} B^{\prime} C^{\prime}$. If not, state how you know they are not similar.

4. Are the two triangles shown below similar? If so, describe a sequence that would prove $\triangle A B C \sim \triangle A^{\prime} B^{\prime} C^{\prime}$. If not, state how you know they are not similar.


## Lesson Summary

A similarity transformation (or a similarity) is a sequence of a finite number of dilations or basic rigid motions. Two figures are similar if there is a similarity transformation taking one figure onto the other figure. Every similarity can be represented as a dilation followed by a congruence.

The notation $\triangle A B C \sim \triangle A^{\prime} B^{\prime} C^{\prime}$ means that $\triangle A B C$ is similar to $\triangle A^{\prime} B^{\prime} C^{\prime}$.

## Problem Set

1. In the picture below, we have triangle $D E F$ that has been dilated from center $O$ by scale factor $r=4$. It is noted by $D^{\prime} E^{\prime} F^{\prime}$. We also have triangle $D^{\prime \prime} E^{\prime \prime} F^{\prime \prime}$, which is congruent to triangle $D^{\prime} E^{\prime} F^{\prime}$ (i.e., $\Delta D^{\prime} E^{\prime} F^{\prime} \cong \Delta D^{\prime \prime} E^{\prime \prime} F^{\prime \prime}$ ). Describe the sequence of a dilation, followed by a congruence (of one or more rigid motions ), that would map triangle $D^{\prime \prime} E^{\prime \prime} F^{\prime \prime}$ onto triangle $D E F$.

2. Triangle $A B C$ was dilated from center $O$ by scale factor $r=\frac{1}{2}$. The dilated triangle is noted by $A^{\prime} B^{\prime} C^{\prime}$. Another triangle $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ is congruent to triangle $A^{\prime} B^{\prime} C^{\prime}$ (i.e., $\Delta A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} \cong \triangle A^{\prime} B^{\prime} C^{\prime}$ ). Describe the dilation followed by the basic rigid motions that would map triangle $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ onto triangle $A B C$.

3. Are the two figures shown below similar? If so, describe a sequence that would prove the similarity. If not, state how you know they are not similar.

4. Triangle $A B C$ is similar to triangle $A^{\prime} B^{\prime} C^{\prime}$ (i.e., $\triangle A B C \sim \triangle A^{\prime} B^{\prime} C^{\prime}$ ). Prove the similarity by describing a sequence that would map triangle $A^{\prime} B^{\prime} C^{\prime}$ onto triangle $A B C$.

5. Are the two figures shown below similar? If so, describe a sequence that would prove $\triangle A B C \sim \Delta A^{\prime} B^{\prime} C^{\prime}$. If not, state how you know they are not similar.

6. Describe a sequence that would show $\triangle A B C \sim \triangle A^{\prime} B^{\prime} C^{\prime}$.


## Lesson 9: Basic Properties of Similarity

## Classwork

## Exploratory Challenge 1

The goal is to show that if $\triangle A B C$ is similar to $\triangle A^{\prime} B^{\prime} C^{\prime}$, then $\triangle A^{\prime} B^{\prime} C^{\prime}$ is similar to $\triangle A B C$. Symbolically, if $\triangle A B C \sim$ $\Delta A^{\prime} B^{\prime} C^{\prime}$, then $\triangle A^{\prime} B^{\prime} C^{\prime} \sim \triangle A B C$.

a. First, determine whether or not $\triangle A B C$ is in fact similar to $\triangle A^{\prime} B^{\prime} C^{\prime}$. (If it isn't, then no further work needs to be done.) Use a protractor to verify that the corresponding angles are congruent and that the ratios of the corresponding sides are equal to some scale factor.
b. Describe the sequence of dilation followed by a congruence that proves $\triangle A B C \sim \triangle A^{\prime} B^{\prime} C^{\prime}$.
c. Describe the sequence of dilation followed by a congruence that proves $\triangle A^{\prime} B^{\prime} C^{\prime} \sim \triangle A B C$.
d. Is it true that $\triangle A B C \sim \triangle A^{\prime} B^{\prime} C^{\prime}$ and $\triangle A^{\prime} B^{\prime} C^{\prime} \sim \triangle A B C$ ? Why do you think this is so?

