## Lesson 1: Why Move Things Around?

## Classwork

## Exploratory Challenge

a. Describe, intuitively, what kind of transformation is required to move the figure on the left to each of the figures (1)-(3) on the right. To help with this exercise, use a transparency to copy the figure on the left. Note: Begin by moving the left figure to each of the locations in (1), (2), and (3).

b. Given two segments $A B$ and $C D$, which could be very far apart, how can we find out if they have the same length without measuring them individually? Do you think they have the same length? How do you check? In other words, why do you think we need to move things around on the plane?


## Lesson Summary

A transformation $F$ of the plane is a function that assigns to each point $P$ of the plane a point $F(P)$ in the plane.

- By definition, the symbol $F(P)$ denotes a specific single point, unambiguously.
- The point $F(P)$ will be called the image of $P$ by $F$. Sometimes the image of $P$ by $F$ is denoted simply as $P^{\prime}$ (read " $P$ prime").
- The transformation $F$ is sometimes said to "move" the point $P$ to the point $F(P)$.
- We also say $F$ maps $P$ to $F(P)$.

In this module, we will mostly be interested in transformations that are given by rules, that is, a set of step-by-step instructions that can be applied to any point $P$ in the plane to get its image.

If given any two points $P$ and $Q$, the distance between the images $F(P)$ and $F(Q)$ is the same as the distance between the original points $P$ and $Q$, and then the transformation $F$ preserves distance, or is distance-preserving.

- A distance-preserving transformation is called a rigid motion (or an isometry), and the name suggests that it moves the points of the plane around in a rigid fashion.


## Problem Set

1. Using as much of the new vocabulary as you can, try to describe what you see in the diagram below.

2. Describe, intuitively, what kind of transformation is required to move Figure $A$ on the left to its image on the right.


## Lesson 8: Sequencing Reflections and Translations

## Classwork

## Exercises 1-3

Use the figure below to answer Exercises 1-3.


1. Figure $A$ was translated along vector $\overrightarrow{B A}$, resulting in Translation(Figure $A$ ). Describe a sequence of translations that would map Figure $A$ back onto its original position.
2. Figure $A$ was reflected across line $L$, resulting in Reflection(Figure $A$ ). Describe a sequence of reflections that would map Figure $A$ back onto its original position.
3. Can Translation $\overrightarrow{B A}$ of Figure $A$ undo the transformation of Translation $\overrightarrow{D C}$ of Figure $A$ ? Why or why not?

## Exercises 4-7

Let $S$ be the black figure.

4. Let there be the translation along vector $\overrightarrow{A B}$ and a reflection across line $L$.

Use a transparency to perform the following sequence: Translate figure $S$; then, reflect figure $S$. Label the image $S^{\prime}$.
5. Let there be the translation along vector $\overrightarrow{A B}$ and a reflection across line $L$.

Use a transparency to perform the following sequence: Reflect figure $S$; then, translate figure $S$. Label the image $S^{\prime \prime}$.
6. Using your transparency, show that under a sequence of any two translations, Translation and Translation ${ }_{0}$ (along different vectors), that the sequence of the Translation followed by the Translation ${ }_{0}$ is equal to the sequence of the Translation followed by the Translation. That is, draw a figure, $A$, and two vectors. Show that the translation along the first vector, followed by a translation along the second vector, places the figure in the same location as when you perform the translations in the reverse order. (This fact is proven in high school Geometry.) Label the transformed image $A^{\prime}$. Now, draw two new vectors and translate along them just as before. This time, label the transformed image $A^{\prime \prime}$. Compare your work with a partner. Was the statement "the sequence of the Translation followed by the Translation $n_{0}$ is equal to the sequence of the Translation Tollowed by the $^{2}$ Translation" true in all cases? Do you think it will always be true?
7. Does the same relationship you noticed in Exercise 6 hold true when you replace one of the translations with a reflection. That is, is the following statement true: A translation followed by a reflection is equal to a reflection followed by a translation?

## Lesson Summary

- A reflection across a line followed by a reflection across the same line places all figures in the plane back onto their original position.
- A reflection followed by a translation does not necessarily place a figure in the same location in the plane as a translation followed by a reflection. The order in which we perform a sequence of rigid motions matters.


## Problem Set

1. Let there be a reflection across line $L$, and let there be a translation along vector $\overrightarrow{A B}$, as shown. If $S$ denotes the black figure, compare the translation of $S$ followed by the reflection of $S$ with the reflection of $S$ followed by the translation of $S$.

2. Let $L_{1}$ and $L_{2}$ be parallel lines, and let Reflection $n_{1}$ and Reflection ${ }_{2}$ be the reflections across $L_{1}$ and $L_{2}$, respectively (in that order). Show that a Reflection ${ }_{2}$ followed by Reflection ${ }_{1}$ is not equal to a Reflection ${ }_{1}$ followed by Reflection $n_{2}$. (Hint: Take a point on $L_{1}$ and see what each of the sequences does to it.)

3. Let $L_{1}$ and $L_{2}$ be parallel lines, and let Reflection ${ }_{1}$ and Reflection $n_{2}$ be the reflections across $L_{1}$ and $L_{2}$, respectively (in that order). Can you guess what Reflection followed by Reflection $_{2}$ is? Give as persuasive an argument as you can. (Hint: Examine the work you just finished for the last problem.)

## Lesson 10: Sequences of Rigid Motions

## Classwork

## Exercises

1. In the following picture, triangle $A B C$ can be traced onto a transparency and mapped onto triangle $A^{\prime} B^{\prime} C^{\prime}$. Which basic rigid motion, or sequence of, would map one triangle onto the other?

2. In the following picture, triangle $A B C$ can be traced onto a transparency and mapped onto triangle $A^{\prime} B^{\prime} C^{\prime}$. Which basic rigid motion, or sequence of, would map one triangle onto the other?

3. In the following picture, triangle $A B C$ can be traced onto a transparency and mapped onto triangle $A^{\prime} B^{\prime} C^{\prime}$. Which basic rigid motion, or sequence of, would map one triangle onto the other?

4. In the following picture, we have two pairs of triangles. In each pair, triangle $A B C$ can be traced onto a transparency and mapped onto triangle $A^{\prime} B^{\prime} C^{\prime}$.
Which basic rigid motion, or sequence of, would map one triangle onto the other?
Scenario 1:


Scenario 2:

5. Let two figures $A B C$ and $A^{\prime} B^{\prime} C^{\prime}$ be given so that the length of curved segment $A C$ equals the length of curved segment $A^{\prime} C^{\prime},|\angle B|=\left|\angle B^{\prime}\right|=80^{\circ}$, and $|A B|=\left|A^{\prime} B^{\prime}\right|=5$. With clarity and precision, describe a sequence of rigid motions that would map figure $A B C$ onto figure $A^{\prime} B^{\prime} C^{\prime}$.


## Problem Set

1. Let there be the translation along vector $\vec{v}$, let there be the rotation around point $A,-90$ degrees (clockwise), and let there be the reflection across line $L$. Let $S$ be the figure as shown below. Show the location of $S$ after performing the following sequence: a translation followed by a rotation followed by a reflection.

2. Would the location of the image of $S$ in the previous problem be the same if the translation was performed last instead of first; that is, does the sequence, translation followed by a rotation followed by a reflection, equal a rotation followed by a reflection followed by a translation? Explain.
3. Use the same coordinate grid to complete parts (a)-(c).

a. Reflect triangle $A B C$ across the vertical line, parallel to the $y$-axis, going through point $(1,0)$. Label the transformed points $A, B, C$ as $A^{\prime}, B^{\prime}, C^{\prime}$, respectively.
b. Reflect triangle $A^{\prime} B^{\prime} C^{\prime}$ across the horizontal line, parallel to the $x$-axis going through point $(0,-1)$. Label the transformed points of $A^{\prime}, B^{\prime}, C^{\prime}$ as $A^{\prime \prime}, B^{\prime \prime}, C^{\prime \prime}$, respectively.
c. Is there a single rigid motion that would map triangle $A B C$ to triangle $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ ?

## Lesson 11: Definition of Congruence and Some Basic Properties

## Classwork

## Exercise 1

a. Describe the sequence of basic rigid motions that shows $S_{1} \cong S_{2}$.

b. Describe the sequence of basic rigid motions that shows $S_{2} \cong S_{3}$.

c. Describe a sequence of basic rigid motions that shows $S_{1} \cong S_{3}$.


## Exercise 2

Perform the sequence of a translation followed by a rotation of Figure $X Y Z$, where $T$ is a translation along a vector $\overrightarrow{A B}$, and $R$ is a rotation of $d$ degrees (you choose $d$ ) around a center $O$. Label the transformed figure $X^{\prime} Y^{\prime} Z^{\prime}$. Is $X Y Z \cong X^{\prime} Y^{\prime} Z^{\prime}$ ?


## Lesson Summary

Given that sequences enjoy the same basic properties of basic rigid motions, we can state three basic properties of congruences:
(Congruence 1) A congruence maps a line to a line, a ray to a ray, a segment to a segment, and an angle to an angle.
(Congruence 2) A congruence preserves lengths of segments.
(Congruence 3) A congruence preserves measures of angles.
The notation used for congruence is $\cong$.

## Problem Set

1. Given two right triangles with lengths shown below, is there one basic rigid motion that maps one to the other? Explain.

2. Are the two right triangles shown below congruent? If so, describe a congruence that would map one triangle onto the other.

3. Given two rays, $\overrightarrow{O A}$ and $\overrightarrow{O^{\prime} A^{\prime}}$ :

a. Describe a congruence that maps $\overrightarrow{O A}$ to $\overrightarrow{O^{\prime} A^{\prime}}$.
b. Describe a congruence that maps $\overrightarrow{O^{\prime} A^{\prime}}$ to $\overrightarrow{O A}$.
